Lagrangian Becchi-Rouet-Stora-Tyutin treatment of collective coordinates

Juan P. Garrahan,* Martín Kruczenski, and Daniel R. Bes

Departamento de Física, Comisión Nacional de Energía Atómica, Av. Libertador 8250, 1429

Buenos Aires, Argentina.

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Abstract

The Becchi-Rouet-Stora-Tyutin (BRST) treatment for the quantization of collective coordinates is considered in the Lagrangian formalism. The motion of a particle in a Riemannian manifold is studied in the case when the classical solutions break a non-abelian global invariance of the action. Collective coordinates are introduced, and the resulting gauge theory is quantized in the BRST antifield formalism. The partition function is computed perturbatively to two-loops, and it is shown that the results are independent of gauge-fixing parameters.

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*Electronic address: garrahan@tandar.cnea.edu.ar.

I. INTRODUCTION

The correct quantization of collective coordinates has been a central issue in the study of soliton models. For example, in the Skyrme model [1] baryons are solitons of an $SU(2)_L \times SU(2)_R/SU(2)_V$ quiral Lagrangian and internal degrees of freedom (spin and isospin) are described as collective excitations of the soliton [2]. These collective excitations typically appear as zero energy fluctuations around a classical solution which breaks certain invariance of the action. To deal with the zero modes the degrees of freedom associated with the collective motion should be isolated. As this is difficult to do in general, collective coordinates can be introduced explicitly as *additional* degrees of freedom. This procedure has become known as the collective coordinate method [3–5].

The general way to introduce collective variables is as parameters of transformations of the original fields [6]. The transformed theory, which depends upon the overcomplete set of original fields plus collective variables, is a gauge theory.

The most rigorous treatment of gauge theories is the Becchi-Rouet-Stora-Tyutin (BRST) quantization procedure [7,8]. The application of the BRST method to the quantization of collective coordinates in finite systems has been developed in [9–12]. The BRST quantization of collective coordinates or fields in the case of field theory was presented in [13]. Simple solitonic models have also been studied using these framework [14,15].

In finite systems and soliton models, collective coordinates are introduced as parameters of the symmetry transformations which are broken (or partially broken) by the classical solutions.[1] The resulting gauge theory is quantized following the usual canonical BRST procedure. Lagrange multipliers and ghosts are introduced for each gauge freedom. An hermitian, nilpotent BRST charge is constructed, and physical states and operators are defined as those invariant under the global BRST symmetry. The gauge theory is thus

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[1]The case in which the transformations are neither restricted to form a group nor to be symmetries of the action is discussed in [13].

replaced by a gauge-fixed BRST theory, in which the collective variables become physical, and the zero modes are cancelled by the Lagrange multipliers and the ghosts.

The BRST quantization can be formulated as a Lagrangian path integral by means of the antifield formalism [16,17]. Given a gauge theory with its corresponding ghosts, antifields are introduced, and an antibracket structure is defined. The gauge-fixed BRST action is obtained from the solution of the so-called Master Equation. Everything is formulated directly in the space of Lagrangian variables.

The purpose of the present paper is to apply the BRST treatment of collective coordinates in the Lagrangian path integral framework to the motion of a particle in a Riemannian manifold. This model includes the Skyrme and the $O(3)$ models, as well as very simple cases which can be used to verify the calculations [18].

Although this model could be treated with the usual Faddev-Popov techniques, the antifield method should be necesary to treat for example supersymmetric solitons when the supersymmetric algebra closes on-shell (i.e. up to terms proportional to equations of motion).

The outline of this paper is as follows. In Sec.II we describe the model and the group G of transformations that leave the action invariant. The classical solution to the equations of motion, as shown in Sec.III, is only invariant under a subgroup H of G, giving rise to zero modes. Collective coordinates have to be introduced to restore the broken symmetry. This yields a gauge invariant action.

In Sec.IV the antifield formalism is applied to the quantization of the gauge invariant action. A BRST gauge-fixed Lagrangian action is constructed, and the path integral for the quantization is set up. An equivalent result can be obtained from the Hamiltonian approach, as mentioned in Sec.V.

Sec.VI discusses the perturbative computation of the partition function. Expressions for the free propagators and interaction vertices are given. Two-loops corrections to the intrinsic energy levels are calculated (formally) with the aid of finite temperature techniques, and a collective Hamiltonian to one loop is obtained. These results are shown to be free of any

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dependence upon spurious parameters introduced by the gauge-fixing procedure.

One aspect not treated in the present paper is the ultraviolet renormalization since it depends on the particular model to which the formalism is applied.

Finally, our conclusions and outlook are given in Sec.VII. The Appendix contains definitions and relations used in calculating the diagrams.

II. THE MODEL

Consider a particle moving in a Riemannian manifold M with metric g_{st}. The Euclidean action is taken to be:

$$S = \int d\tau \left(\frac{1}{2} g_{st} \dot{q}^s \dot{q}^t + V[q] \right), \tag{1}$$

The coordinates q^s parametrize the manifold and $V[q]$ is an arbitrary potential. The dot $\dot{q}^s$ denotes derivatives respect to Euclidean time τ. Summation is implicit over repeated indices $s, t, \ldots$, summation over other indices are written explicitly.

A point q^s in M describes a configuration of the system. For example in the Skyrme model M is the (infinite dimensional) manifold of functions (of winding number one) q : $S^3 \rightarrow SU(2)$, where S^3 is the compactified physical space and $SU(2)$ is parametrized by the pions. In this case the variable q is a scalar field $\phi_a(\vec{x}), a = 1, 2, 3$. In the following, s stands for both a and $\vec{x}$. Sums in s are then understood as sums over a and integrals over $\vec{x}$. With this conventions, our formalism can be applied to soliton models as well as to simple quantum mechanical models.

We assume that there exists an unbroken non-trivial, finite-dimensional, group G of symmetries of the action, constituted by isometries which also leave the potential invariant:

$$q^s \rightarrow R^s(q, \alpha_a), \tag{2}$$

$$g_{st}(q) = \partial_s R^u \partial_t R^v g_{uv}(R(q, \alpha_a)), \tag{3}$$

$$V[q] = V[R(q, \alpha_a)], \tag{4}$$

where the α_a's parametrize the group G. By ∂_s and ∂_a we mean $\frac{\partial}{\partial q^s}$ and $\frac{\partial}{\partial \alpha_a}$. The parametrization is such that $\alpha_a = 0$ corresponds to the identity $(R^s(q, 0) = q^s)$.

Infinitesimal variations, which generate the Lie algebra $\mathbf{g}$ of G are defined as:

$$\delta_a q^s = \partial_a R^s(q, \alpha)|_{\alpha=0}. \tag{5}$$

Commuting two such infinitesimal variations we obtain:

$$\partial_a R^t \partial_b \partial_t R^s - \partial_a \partial_t R^s \partial_b R^t = C_{ab}{}^c \partial_c R^s, \tag{6}$$

where $C_{ab}{}^c$ are the structure constants of the group G.

In the next section a semiclassical expansion is performed around a minimum of the potential V. The interesting case is that in which there are several classical minima related by transformations of G whereas quantum mechanically there is only a single vacuum because G is unbroken (this is not the case of spontaneous symmetry breaking in which there are many *quantum* vacua). Choosing a particular classical minimum brings in infrared divergencies which spoil the calculation (excitations tangent to the surface of minima have zero restoration frequency). The well known cure is the introduction of collective coordinates which eliminate the zero modes and at the same time restore the symmetry which was broken at the classical level.

III. STATIC SOLUTIONS AND COLLECTIVE COORDINATES

The equations of motion obtained from (1) are

$$g_{st} \ddot{q}^t + \frac{1}{2} \dot{q}^t \dot{q}^u \left(\partial_u g_{st} + \partial_t g_{su} - \partial_s g_{tu} \right) - \partial_s V = 0. \tag{7}$$

Static solutions satisfy the time-independent equation

$$\partial_s V|_{q=\bar{q}} = 0. \tag{8}$$

We choose a particular static solution $\bar{q}$ and expand the field around it, $q \to \bar{q} + q$. The linearised equations for the fluctuations read

$$\left(\bar{V}_{st} - \omega^2 \bar{g}_{st} \right) q^t = 0, \tag{9}$$

where

$$\bar{g}_{st,u...v} = \partial_u \cdots \partial_v g_{st}\big|_{q=\bar{q}} \ , \qquad \bar{V}_{u...v} = \partial_u \cdots \partial_v V\big|_{q=\bar{q}} \ . \tag{10}$$

The set of infinitesimal transformations $\{\delta_a\}$ is split into two sets, $\{\delta_{a'}\}$ which change the static solutions, and $\{\delta_{\bar{a}}\}$ which leave them invariant. The elements of G which leave the minimum invariant form a compact subgroup H of G with Lie algebra $\mathbf{h}$ [19]. The parametrization α can be chosen such that the generators of $\mathbf{g} - \mathbf{h}$ and $\mathbf{h}$, $\{\delta_{a'}\}$ and $\{\delta_{\bar{a}}\}$ respectively, are ortogonal to each other. Furthermore, we make the simplifying assumption that G/H is a symmetric space which is valid for the Skyrme and $O(3)$ models. These conditions are resumed in the following relations:

$$[\mathbf{g} - \mathbf{h}, \mathbf{h}] \subset \mathbf{g} - \mathbf{h}, \tag{11}$$

$$[\mathbf{h}, \mathbf{h}] \subset \mathbf{h}, \tag{12}$$

$$[\mathbf{g} - \mathbf{h}, \mathbf{g} - \mathbf{h}] \subset \mathbf{h}, \tag{13}$$

Therefore, the only non-zero structure constants are $C_{a'\bar{b}}{}^{c'}$, $C_{\bar{a}\bar{b}}{}^{\bar{c}}$ and $C_{a'b'}{}^{\bar{c}}$.

The generators of $\mathbf{g} - \mathbf{h}$ give rise to zero modes

$$\bar{V}_{st}\psi_{a'}^t = 0, \tag{14}$$

$$\psi_{a'}^s = \Im_{a'}^{-1/2}\delta_{a'}\bar{q}^s, \tag{15}$$

$$\Im_{a'} = \bar{g}_{st}\delta_{a'}\bar{q}^s\delta_{a'}\bar{q}^t. \tag{16}$$

The set of normal modes $\{\psi_n^s\} = \{\psi_{\bar{n}}^s; \psi_{a'}^s\}$,

$$\bar{V}_{st}\psi_{a'}^t = 0 \ , \qquad \left(\bar{V}_{st} - \omega_{\bar{n}}^2 \bar{g}_{st} \right) \psi_{\bar{n}}^t = 0 \quad (\omega_{\bar{n}} \neq 0), \tag{17}$$

satisfies the orthogonality and completeness relations:

$$\bar{g}_{st}\psi_{\bar{n}}^s\psi_{\bar{m}}^t = \delta_{\bar{n}\bar{m}} \ , \qquad \bar{g}_{st}\psi_{a'}^s\psi_{b'}^t = \delta_{a'b'} \ , \qquad \bar{g}_{st}\psi_{\bar{n}}^s\psi_{a'}^t = 0, \tag{18}$$

$$\sum_{\bar{n}} \bar{g}_{st}\psi_{\bar{n}}^t\psi_{\bar{n}}^u + \sum_{a'} \bar{g}_{st}\psi_{a'}^t\psi_{a'}^u = \delta_s^u. \tag{19}$$

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We have assumed that the group G is unbroken at quantum level, so the vacuum must be invariant under G. Since the static solution is not, we restore the symmetry by the usual procedure of including collective coordinates.

Now we perform a time-dependent transformation of the fields q [6,12]

$$q^s \to R^s(q, \alpha(t)). \tag{20}$$

Care must be taken when performing this kind of transformations in the path integral because extra terms of order $\hbar^2$ may arise [20]. With the choice of gauge we make below they are absent, but in other gauges must be included [21]. In the context of collective coordinates quantization these terms were discussed in [22].

The transformed action reads

$$S'_0 = \int d\tau \left[\frac{1}{2} g_{st} \left(\dot{q}^s + \sum_{a,b} \dot{\alpha}_a \zeta_{ab} \delta_b q^s \right) \left(\dot{q}^t + \sum_{c,d} \dot{\alpha}_c \zeta_{cd} \delta_d q^t \right) + V[q] \right], \tag{21}$$

where the matrix ζ_{ab} is defined by means of the identity

$$\frac{\partial q^s}{\partial q'^t} \partial_a R^t(q, \alpha) = \sum_b \zeta_{ab}(\alpha) \partial_b R^s(q, 0), \tag{22}$$

where $q'^s = R^s(q, \alpha)$. This is to be interpreted as follows: the inverse of R maps q' into q, and the differential of R^{-1}, maps the tangent space to M in q' $(TM_{q'})$ into the tangent space in q (TM_q). $\partial_a R^s(q, \alpha)$ belongs to $TM_{q'}$ and is mapped into $\zeta_{ab} \partial_b R^s(q, 0)$ which belongs to TM_q.

We now consider the parameters α_a as genuine (collective) variables of the problem. It is easy to show that the action S'_0 is invariant under gauge transformations [6,12]

$$\begin{pmatrix} q^s \\ \alpha_b \end{pmatrix} \to \exp \sum_a \epsilon_a(\tau) \delta_a \begin{pmatrix} q^s \\ \alpha_b \end{pmatrix}, \tag{23}$$

where

$$\delta_a \alpha_b = -(\zeta^{-1})_{ab}. \tag{24}$$

The above system may be interpreted as describing the problem from a moving frame of reference oriented according to the collective variables, and moving with velocity $\dot{\alpha}_a \zeta_{ab}$ along

the direction b. The gauge invariance is a manifestation of the fact that transforming the intrinsic coordinates and correspondingly moving the frame of reference leaves the physical situation unchanged.

IV. ANTIFIELD FORMALISM

The antifield formalism or Batalin-Vilkovisky formalism [16] provides a general method for the quantization of gauge theories within a Lagrangian framework. The BRST transformation [23] and a canonical formalism are defined in the space of Lagrangian variables by the introduction of ghosts and antifields. A gauge-fixed quantum action is obtained from an equation (Master Equation) plus boundary conditions. The procedure is algebraic and straightforward, but nevertheless powerful. The antifield formalism is reviewed in [8,17].

In this section we apply the antifield formalism to the problem presented in the previous sections. We start from the action $S_0'[q, \alpha]$ (21) which is invariant under gauge transformations (23). The set of variables $\{q^s, \alpha_a\}$ is enlarged by the introduction, for each of the independent gauge generators, of (antihermitian) bosonic variables b_a, and (hermitian) fermionic ghosts pairs η_a and $\bar{\eta}_a$. The set of all fields and coordinates will be generically denoted as $\{\phi_A\} = \{q^s, \alpha_a, b_a, \eta_a, \bar{\eta}_a\}$.

For each of the fields[2] $\{\phi_A\}$ an *antifield* $\{\phi_A^*\}$ is introduced, with opposite Grassmann parity and hermiticity. The doubling of the fields allows for the definition of a bracket structure, called the antibracket. The antibracket of arbitrary functionals of the fields and antifields is defined as

$$(F, G) = \int d\tau \left(\frac{\delta^R F}{\delta \phi_A(\tau)} \frac{\delta^L G}{\delta \phi_A^*(\tau)} - \frac{\delta^R F}{\delta \phi_A^*(\tau)} \frac{\delta^L G}{\delta \phi_A(\tau)} \right), \tag{25}$$

where the superscript R (L) stands for right (left) derivative which differ when deriving

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[2]Following the literature we denote such quantities by fields, although some of them are actually variables in our case.

respect to a fermion. The antibracket of fields and antifields plays an analogous role to the Poisson bracket, because within this structure fields are conjugate to antifields.

The next step is to find an action $S[\phi_A, \phi_A^*]$ which satisfies the Master Equation $(S, S) = 0$ with the boundary condition that it becomes equal to the original action when the antifields vanish, i.e., $S[\phi_A, \phi_A^* = 0] = S_0'[q, \alpha]$. It can be shown [8,17] that this requirements are satisfied by[3]

$$S = S' + \int d\tau \left(\sum_a q_s^* \delta_a q^s \eta_a - \sum_{ab} \alpha_b^* \left(\zeta^{-1} \right)_{ab} \eta_a + \sum_{abc} \frac{1}{2} C_{ab}{}^c \eta_c^* \eta_a \eta_b + \sum_a \bar{\eta}_a^* b_a \right). \tag{26}$$

S carries the information relative to the gauge algebra of the problem through the presence of the structure constants. It is the starting action to quantize the theory, for which a gauge fixing procedure must be implemented.

The gauge fixed action is defined as

$$S_\psi \equiv S \left[\phi_A, \phi_A^* = \frac{\delta\psi}{\delta\phi_A} \right], \tag{27}$$

where ψ is an imaginary fermionic function of the fields only.[4] The path integral for the quantization of the classical theory S' is given by

$$Z = \int \mathcal{D}[q, \eta, \bar{\eta}, b] \mathcal{D}[\alpha] |\zeta| \sqrt{g} \exp(-S_\psi[q^s, \alpha_a, \eta_a, \bar{\eta}_a, b_a]), \tag{28}$$

where $\mathcal{D}[\alpha]|\zeta|$ is the measure over the group manifold, and $\mathcal{D}[q]\sqrt{g} = \mathcal{D}[q]\sqrt{\det(g_{st})}$ is the measure over M, which can be exponentiated as

$$\int \mathcal{D}[q]\sqrt{g}e^{-S_\psi} = \int \mathcal{D}[q] \prod_\tau \sqrt{g(\tau)} e^{-S_\psi} = \int \mathcal{D}[q] \exp \left(\frac{\delta(0)}{2} \int d\tau g(\tau) - S_\psi \right). \tag{29}$$

We make the following choice for the gauge fixing fermion

$$\psi = -i \int \left\{ \sum_{a'} \left[\left(\omega_{a'}^2 G_{a'} + \sum_b \frac{\partial}{\partial \tau} \left(\dot{\alpha}_b \zeta_{ba'} \right) \right) + \frac{i\omega_{a'}^2}{2\Im_{a'}} b_{a'} \right] \bar{\eta}_{a'} + \sum_{\bar{a},b} \dot{\alpha}_b \zeta_{b\bar{a}} \bar{\eta}_{\bar{a}} \right\}, \tag{30}$$

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[3]To check $(S, S) = 0$ use is made of the group identity $\frac{\partial}{\partial\alpha_c}\zeta_{ab} - \frac{\partial}{\partial\alpha_a}\zeta_{cb} + \zeta_{af}\zeta_{cd}C_{fd}{}^b = 0$.

[4]ψ is called the gauge fixing fermion.

where the functions $G_{a'}$ depend on the fields q, and the inertia parameters $\Im_{a'}$ have been defined in (16). The parameters $\omega_{a'}$ are arbitrary and should disappear from any physical (gauge-invariant) result. They will be interpreted as the frequencies associated with the spurious sector (cf. Eqs.(58-60)). The gauge fixed action reads

$$
\begin{aligned}
S_\psi = S' + \int \Bigg\{ &\sum_{a'} \left[\frac{\omega_{a'}^2}{2\Im_{a'}} b_{a'} b_{a'} - i b_{a'} \left(\omega_{a'}^2 G_{a'} + \sum_b \frac{\partial}{\partial \tau} (\dot{\alpha}_b \zeta_{ba'}) \right) - i \dot{\bar{\eta}}_{a'} \dot{\eta}_{a'} \right] \\
&+ \sum_{\bar{a}} \left[-i b_{\bar{a}} \left(\sum_b \dot{\alpha}_b \zeta_{b\bar{a}} \right) + i \bar{\eta}_{\bar{a}} \dot{\eta}_{\bar{a}} \right] - \sum_{a',b} i \omega_{a'}^2 \bar{\eta}_{a'} (\delta_b G_{a'}) \eta_b \\
&- \sum_{a',b,c'} i C_{a'b}{}^{c'} \left(\sum_d \dot{\alpha}_d \zeta_{da'} \right) \eta_b \dot{\bar{\eta}}_{c'} + \sum_{a,b,\bar{c}} i C_{ab}{}^{\bar{c}} \left(\sum_d \dot{\alpha}_d \zeta_{da} \right) \eta_b \bar{\eta}_{\bar{c}} \Bigg\}.
\end{aligned}
\tag{31}
$$

The b fields can be integrated out. Integration over $b_{\bar{a}}$ gives delta functions, and those over $b_{a'}$ are gaussian which can be interpreted as averages over gauge fixing delta functions:

$$
Z = \int \mathcal{D}[q, \alpha, \eta, \bar{\eta}] |\zeta| \sqrt{g} \prod_{\bar{a}} \delta \left(\sum_b \dot{\alpha}_b \zeta_{b\bar{a}} \right) \exp(-S_\psi),
\tag{32}
$$

$$
\begin{aligned}
S_\psi = S' + \int \Bigg\{ &\sum_{a'} \left[\frac{\Im_{a'}}{2\omega_{a'}^2} \left(\omega_{a'}^2 G_{a'} + \sum_b \frac{\partial}{\partial \tau} (\dot{\alpha}_b \zeta_{ba'}) \right) \times \right. \\
&\times \left(\omega_{a'}^2 G_{a'} + \sum_b \frac{\partial}{\partial \tau} (\dot{\alpha}_b \zeta_{ba'}) \right) - i \dot{\bar{\eta}}_{a'} \dot{\eta}_{a'} \right] + \sum_{\bar{a}} i \bar{\eta}_{\bar{a}} \dot{\eta}_{\bar{a}} \\
&- \sum_{a',b} i \omega_{a'}^2 \bar{\eta}_{a'} (\delta_b G_{a'}) \eta_b - \sum_{a',b,c',d} i C_{a'b}{}^{c'} (\dot{\alpha}_d \zeta_{da'}) \eta_b \dot{\bar{\eta}}_{c'} + \sum_{a,b,\bar{c},d} i C_{ab}{}^{\bar{c}} (\dot{\alpha}_d \zeta_{da}) \eta_b \bar{\eta}_{\bar{c}} \Bigg\}.
\end{aligned}
\tag{33}
$$

To get rid of the second order derivatives $\ddot{\alpha}$ let us introduce auxiliary coordinates $\lambda_{a'}$ in the path integral for S_ψ

$$
\begin{aligned}
Z &= \int \mathcal{D}[q, \alpha, \eta, \bar{\eta}] |\zeta| \sqrt{g} \prod_{\bar{a}} \delta \left(\sum_b \dot{\alpha}_b \zeta_{b\bar{a}} \right) \exp(-S_\psi) \\
&= \int \mathcal{D}[q, \alpha, \eta, \bar{\eta}, \lambda] |\zeta| \sqrt{g} \prod_{a'} \delta \left(\frac{\omega_{a'}}{\sqrt{\Im_{a'}}} \lambda_{a'} - \sum_b \dot{\alpha}_b \zeta_{ba'} \right) \prod_{\bar{a}} \delta \left(\sum_b \dot{\alpha}_b \zeta_{b\bar{a}} \right) \exp(-S_\psi)
\end{aligned}
\tag{34}
$$

The delta functions can be exponentiated by means of fields $\mathcal{P}$. We obtain

$$
Z = \int \mathcal{D}[q, \alpha, \mathcal{P}, \lambda, \eta, \bar{\eta}] \sqrt{g} \exp(-S_\psi),
\tag{35}
$$

$$
S'_\psi = S_{\text{intr.}} + S_{\text{coll.}} + S_{\text{coup.}},
\tag{36}
$$

$$
S_{\text{intr.}} = \int d\tau \left[\frac{1}{2} g_{st} \left(\dot{q}^s + \sum_{a'} \frac{\omega_{a'}}{\sqrt{\Im_{a'}}} \lambda_{a'} \delta_{a'} q^s \right) \left(\dot{q}^t + \sum_{b'} \frac{\omega_{b'}}{\sqrt{\Im_{b'}}} \lambda_{b'} \delta_{b'} q^t \right) + V[q] \right.
$$

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$$-\sum_{a'} \frac{\Im_{a'}}{2\omega_{a'}^2} \left(\omega_{a'}^2 G_{a'} - \frac{\omega_{a'}}{\sqrt{\Im_{a'}}} \dot{\lambda}_{a'}\right) \left(\omega_{a'}^2 G_{a'} - \frac{\omega_{a'}}{\sqrt{\Im_{a'}}} \dot{\lambda}_{a'}\right)$$

$$-\sum_{a'} i\dot{\bar{\eta}}_{a'} \dot{\eta}_{a'} + \sum_{\bar{a}} i\bar{\eta}_{\bar{a}} \dot{\eta}_{\bar{a}} - \sum_{a',b} i\omega_{a'}^2 \bar{\eta}_{a'} \left(\delta_b G_{a'}\right) \eta_b$$

$$\left. -\sum_{a',b,c'} iC_{a'b}{}^{c'} \frac{\omega_{a'}}{\sqrt{\Im_{a'}}} \lambda_{a'} \eta_b \dot{\bar{\eta}}_{c'} + \sum_{a',b,\bar{c}} iC_{a'b}{}^{\bar{c}} \frac{\omega_{a'}}{\sqrt{\Im_{a'}}} \lambda_{a'} \eta_b \bar{\eta}_{\bar{c}} \right] \tag{37}$$

$$S_{\text{coll.}} = -i \int d\tau \sum_a \dot{\alpha}_a \mathcal{P}_a, \tag{38}$$

$$S_{\text{coup.}} = \frac{i\omega}{a} \int d\tau \sum_{a'b} \frac{\omega_{a'}}{\sqrt{\Im_{a'}}} \lambda_{a'} (\zeta^{-1})_{a'b} \mathcal{P}_b. \tag{39}$$

The action for the *intrinsic* variables (original fields q, Lagrange multipliers λ and ghosts $\eta, \bar{\eta}$) is $S_{\text{intr.}}$. It contains the transformed action (21), but with the Lagrange multipliers as the velocities of the moving frame. It also has a gauge fixing term, the action for the ghosts, and the coupling between the ghosts and the bosonic variables.

$S_{\text{coll.}}$ is the free action for the collective coordinates, in Hamiltonian form. From it we see that $\mathcal{P}_a = \delta S_{coll.}/\delta\dot{\alpha}_a$ is to be interpreted as the canonical momenta conjugate to the angles α_a. Therefore $\mathcal{D}[\alpha, \mathcal{P}]$ is the canonical invariant phase space measure. The mixed Lagrangian-Hamiltonian form is frequent in collective coordinate problems [24].

The coupling between collective and intrinsic degrees of freedom is given by $S_{\text{coup.}}$.

V. THE HAMILTONIAN BRST QUANTIZATION

In the previous section the gauge-fixed BRST Lagrangian path integral was constructed by means of the antifield formalism. A similar result can be obtained by integrating out the momenta in a Hamiltonian BRST path integral. In this section we outline such a procedure to show its equivalence to the antifield approach. Hamiltonian BRST quantization is reviewed in Ref. [8]. Its application to collective coordinates can be found in [12].

The classical Hamiltonian corresponding to the gauge invariant Lagrangian action (21) is

$$H = \frac{1}{2} g^{st} p_s p_t + V[q], \tag{40}$$

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plus a set of abelian first-class constraints $F_a = \sum_b \zeta_{ab} j_b - \mathcal{P}_a$, with Poisson brackets $\{p_s, q^t\} = -\delta_s^t$, $\{\mathcal{P}_a, \alpha_b\} = -\delta_{ab}$, and $j_a = p_s \delta_a q^s$. The quantum Hamiltonian has an ordering ambiguity which is saved as usual by taking the kinetic part to be the laplacian over M. A new set of non-abelian constraints can be obtained by multiplying F by ζ^{-1} [12]

$$f_a = \sum_b (\zeta^{-1})_{ab} F_b = j_a - I_a , \qquad \{f_a, f_b\} = -\sum_c C_{ab}{}^c f_c. \tag{41}$$

The operators $I_a = \sum_b (\zeta^{-1})_{ab} \mathcal{P}_b$ satisfy $\{I_a, I_b\} = \sum_c C_{ab}{}^c I_c$, and should be considered as the collective version of the intrinsic operators j_a.

The phase space is enlarged by introducing, for each first-class constraint, a Lagrange multiplier λ_a and two (fermionic) ghosts $\eta_a, \bar{\eta}_a$, together with their corresponding conjugate momenta $b_a, \pi_a, \bar{\pi}_a$. Upon quantization, physical states are defined as those annihilated by the BRST charge Ω

$$\Omega \equiv \sum_a (b_a \bar{\pi}_a - f_a \eta_a) - \frac{i}{2} \sum_{a,b,c} C_{ab}{}^c \eta_a \eta_b \pi_c, \tag{42}$$

and physical operators as those that commute with Ω.

Any term of the form $[\rho, \Omega]$ may be added to the Hamiltonian without altering the overlaps of the original Hamiltonian within the subspace annihilated by Ω. After rescaling the variables: $\lambda_{a'} \to i\omega_{a'} \Im_{a'}^{-1/2} \lambda_{a'}$, $b_{a'} \to -i\Im_{a'}^{1/2} \omega_{a'}^{-1} b_{a'}$, we choose [12]

$$\rho = \sum_{a'} \left[\frac{i\omega_{a'}}{\sqrt{\Im_{a'}}} \lambda_{a'} \pi_{a'} + \left(\omega_{a'}^2 G_{a'} + \frac{i}{2} b_{a'} \right) \bar{\eta}_{a'} \right], \tag{43}$$

which yields, after integrating the momenta p, b, π and $\bar{\pi}$ in the phase space path integral, the Lagrangian path integral of Eqs.(35)-(36).

The functions G are chosen such that $[j_{a'}, G_{b'}] \neq 0$ at leading order, so that the ghost propagators are well-defined. In that case, the BRST Hamiltonian does not commute with the operators j_a. This is desirable for the $j_{a'}$'s, since they give rise to the zero modes, as seen in Sec.III. On the other hand, the symmetry under the transformations $j_{\bar{a}}$ was not broken by the classical solution, and it is convenient to choose a gauge-fixing scheme that conserves such a symmetry. First we must define transformation operators similar to f_a

(which transform the original and collective variables) for the Lagrange multipliers and the ghosts [12]:

$$N_a \equiv -\sum_{bc} C_{ab}{}^c \frac{\omega_{b'} \sqrt{\Im_{c'}}}{\omega_{c'} \sqrt{\Im_{b'}}} \lambda_b b_c, \tag{44}$$

$$\tau_a \equiv i \sum_{bc} C_{ab}{}^c \eta_b \pi_c, \tag{45}$$

$$\bar{\tau}_a \equiv i \sum_{bc} C_{ab}{}^c \bar{\pi}_b \bar{\eta}_c. \tag{46}$$

The above operators have the same commutation relations as the j's.

Transformations of all the variables are generated by the operators

$$L_a \equiv f_a + N_a + \tau_a + \bar{\tau}_a, \tag{47}$$

$$[L_a, L_b] = -\sum_{bc} C_{ab}{}^c L_c, \tag{48}$$

which are null operators [12]

$$L_a = [\Omega, -\sum_{bc} C_{ab}{}^c \frac{i\omega_{b'}}{\sqrt{\Im_{b'}}} \lambda_b \bar{\eta}_c - \pi_a], \tag{49}$$

so they commute with Ω, and map physical states into null (zero norm) states [8]. They also commute with the original Hamiltonian.

The operators $L_{a'}$ do not commute with the gauge-fixing fermion due to the functions $G_{a'}$. However, this functions can be chosen to transform under the $L_{\bar{a}}$'s as,

$$i[L_{\bar{a}}, G_{b'}] = i[j_{\bar{a}}, G_{b'}] = -C_{\bar{a}c'}{}^{b'} G_{c'}. \tag{50}$$

This is indeed the choice made in Sec.VI (Eq.53).

If the arbitrary parameters $\omega_{a'}$ are all taken equal[5] then the operators $L_{\bar{a}}$ commute with the BRST Hamiltonian. Thus the eigenstates of the Hamiltonian can be classified by irreducible representations of the group H. However, in an irreducible representation any state can be obtained from any other by repeated application of the $L_{\bar{a}}$'s, which are null operators. This means that the only representation which is not neccesarily composed by

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[5]In fact they need only be taken equal within each irreducible representation of H in $\mathbf{g} - \mathbf{h}$

null-states is the trivial one in which there is only one state satisfying $L_{\bar{a}}|\rangle = 0$. Physical states belong to this representation.

The action of collective operators $I_{\bar{a}}$ on physical states is determined by the intrinsic structure, i.e.

$$L_{\bar{a}}|\text{ph}\rangle = 0 \Rightarrow I_{\bar{a}}|\text{ph}\rangle = (j_{\bar{a}} + N_{\bar{a}} + \tau_{\bar{a}} + \bar{\tau}_{\bar{a}})|\text{ph}\rangle. \tag{51}$$

Thus the collective operators $I_{\bar{a}}$ can be identified with the intrinsic operators $j_{\bar{a}} + N_{\bar{a}} + \tau_{\bar{a}} + \bar{\tau}_{\bar{a}}$ when taking matrix elements between physical states. This fact is used in the next section when the partition function of the system is calculated.

VI. THE PERTURBATIVE TREATMENT

In this section we evaluate perturbatively the partition function (35), i.e., the trace over physical states of $e^{-\beta H}$. It is known [25] that this amounts to perform the functional integration over fields with periodic boundary conditions in the interval $[0, \beta]$, including the ghosts despite the fact that they are fermions.

Minimization of the action (36) with the assumption that Lagrange multipliers and ghosts vanish classically gives that the only non-vanishing expectation values are those of Eq.(8). Expanding all the fields as fluctuations around their classical values allows to perform a perturbative calculation. The quadratic Lagrangian provides the free propagators, and the third or higher order terms give vertices to be used in the usual Feynman diagram expansion.

To ease the calculation we expand the fluctuations of q^s in terms of the normal modes

$$q^s = \bar{q}^s + \sum_{\bar{n}} \xi_{\bar{n}} \psi_{\bar{n}}^s + \sum_{a'} \xi_{a'} \psi_{a'}^s, \tag{52}$$

where ξ_n are normal coordinates. We choose the gauge fixing functions $G_{a'}$ to be

$$G_{a'} = \Im_{a'}^{-1/2} g_{st} \psi_{a'}^s (q^t - \bar{q}^t) = \Im_{a'}^{-1/2} \xi_{a'}. \tag{53}$$

Such choice cancels at quadratic level the coupling between λ and ξ, and provides (spurious) frequencies to the zero modes and to the ghosts. This gauge is known as 't Hooft gauge [26], or as rigid gauge in soliton problems.

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Now the quadratic action can be written,

$$S_{\text{int.}}^{(2)} = \int d\tau \left(\sum_{\bar{n}} \frac{1}{2} \left(\dot{\xi}_{\bar{n}} \dot{\xi}_{\bar{n}} + \omega_{\bar{n}}^2 \xi_{\bar{n}} \xi_{\bar{n}} \right) + \sum_{a'} \frac{1}{2} \left(\dot{\xi}_{a'} \dot{\xi}_{a'} + \omega_{a'}^2 \xi_{a'} \xi_{a'} + \dot{\lambda}_{a'} \dot{\lambda}_{a'} + \omega_{a'}^2 \lambda_{a'} \lambda_{a'} \right) \right.$$
$$\left. - i \sum_{a'} \left(\dot{\bar{\eta}}_{a'} \dot{\eta}_{a'} + \omega_{a'}^2 \bar{\eta}_{a'} \eta_{a'} \right) + i \sum_{\bar{a}} \bar{\eta}_{\bar{a}} \dot{\eta}_{\bar{a}} \right). \tag{54}$$

To evaluate the partition function to leading order we use the gaussian integrals over periodic time τ

$$\begin{aligned}
\int \mathcal{D}[\phi] \exp \left[-\tfrac{1}{2} \int d\tau (\dot{\phi}^2 + \omega^2 \phi^2) \right] &= \det^{-1/2}(\partial_\tau^2 - \omega^2) \quad \text{(bosonic)}, \\
\int \mathcal{D}[\bar{\eta}\eta] \exp \left[-\int d\tau (\dot{\bar{\eta}}\dot{\eta} + \omega^2 \bar{\eta}\eta) \right] &= \det(\partial_\tau^2 - \omega^2) \quad \text{(fermionic)}.
\end{aligned} \tag{55}$$

We obtain [27]:

$$\begin{aligned}
Z^{(1)} &= \int \mathcal{D}[\xi_n, \lambda_{a'}, \bar{\eta}_{a'}, \eta_{a'}] \sqrt{g} \exp(-S_{\text{intr.}}^{(2)}) \\
&= \prod_{\bar{n}} \det^{-1/2}(\partial_\tau^2 - \omega_{\bar{n}}^2) \prod_{a'} \det^{-1/2}(\partial_\tau^2 - \omega_{a'}^2) \det^{-1/2}(\partial_\tau^2 - \omega_{a'}^2) \det(\partial_\tau^2 - \omega_{a'}^2) \\
&= \prod_{\bar{n}} \left[2 \sinh \left(\frac{\beta \omega_{\bar{n}}}{2} \right) \right]^{-1},
\end{aligned} \tag{56}$$

where β is the inverse temperature. As usual, the ghost determinant cancels out the contribution from the bosonic spurious degrees of freedom $(\lambda_{a'}, \xi_{a'})$. Expanding the above result the usual contribution $\sum_{\bar{n}} \frac{\omega_n}{2}$ to the vacuum energy is found.

The finite temperature propagators can be similarly calculated

$$\langle\langle T \xi_{\bar{n}}(\tau) \xi_{\bar{n}}(\tau') \rangle\rangle = G_{\omega_{\bar{n}}}(\tau - \tau'), \tag{57}$$

$$\langle\langle T \xi_{a'}(\tau) \xi_{a'}(\tau') \rangle\rangle = G_{\omega_{a'}}(\tau - \tau'), \tag{58}$$

$$\langle\langle T \lambda_{a'}(\tau) \lambda_{a'}(\tau') \rangle\rangle = G_{\omega_{a'}}(\tau - \tau'), \tag{59}$$

$$\langle\langle T \bar{\eta}_{a'}(\tau) \eta_{a'}(\tau') \rangle\rangle = -i G_{\omega_{a'}}(\tau - \tau'), \tag{60}$$

where

$$G_\omega(\tau) = \frac{1}{2\omega (e^{\beta\omega} - 1)} \left(e^{\beta\omega} e^{-\omega|\tau|} + e^{\omega|\tau|} \right). \tag{61}$$

The time order in the propagator for two velocities is defined as

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$$\langle\langle \hat{T} \dot{\xi}_n(\tau) \dot{\xi}_n(\tau') \rangle\rangle = \partial_\tau \partial_{\tau'} G_{\omega_n}(\tau - \tau')$$

$$= \delta(\tau - \tau') - \omega_n^2 G_{\omega_n}(\tau - \tau'). \tag{62}$$

The $\delta(\tau - \tau')$ will cancel extra vertices in the Lagrangian (respect to the interaction Hamiltonian), as well as the $\delta(0)$ contributions arising from the measure $\frac{1}{2}\delta(0)\int d\tau g(\tau)$ (Eq.(29)) [28,29].

The higher than quadratic terms in the Lagrangian provide the vertices for constructing Feynman diagrams. The cubic and quartic vertices are given in Fig.1. The straight lines correspond to the fields q, the wiggy lines to λ, and the broken lines to ghosts. The values for the vertices of Fig.1 are (hereafter summation is implicit over any repeated index):

$$(a) = -\frac{1}{6} A_{nml} \xi_n \xi_m \xi_l, \tag{63}$$

$$(b) = -\frac{1}{2} B_{nm;l} \dot{\xi}_n \dot{\xi}_m \xi_l, \tag{64}$$

$$(c) = -\omega_{a'} \left(B_{a'm;n} + D_{mn}^{a'} \right) \lambda_{a'} \xi_n \dot{\xi}_m, \tag{65}$$

$$(d) = -\frac{1}{2} \omega_{a'} \omega_{b'} \left(B_{a'b';n} + D_{b'n}^{a'} + D_{a'n}^{b'} \right) \lambda_{a'} \lambda_{b'} \xi_n, \tag{66}$$

$$(e) = i\omega_{a'}^2 \sqrt{\frac{\Im_{b'}}{\Im_{a'}}} D_{a'n}^{b'} \bar{\eta}_{a'} \xi_n \eta_{b'} + i\omega_{a'}^2 \Im_{a'}^{-\frac{1}{2}} D_{a'n}^{\bar{b}} \bar{\eta}_{a'} \xi_n \eta_{\bar{b}}, \tag{67}$$

$$(f) = -i\omega_{c'} \Im_{c'}^{-\frac{1}{2}} C_{c'\bar{b}}{}^{a'} \dot{\bar{\eta}}_{a'} \lambda_{c'} \eta_{\bar{b}}, \tag{68}$$

$$(g) = i\omega_{c'} \Im_{c'}^{-\frac{1}{2}} C_{c'b'}{}^{\bar{a}} \bar{\eta}_{\bar{a}} \lambda_{c'} \eta_{b'}, \tag{69}$$

$$(h) = -\frac{1}{24} E_{nmlp} \xi_n \xi_m \xi_l \xi_p, \tag{70}$$

$$(i) = -\frac{1}{4} F_{nm,lp} \dot{\xi}_n \dot{\xi}_m \xi_l \xi_p, \tag{71}$$

$$(j) = -\omega_{a'} \left(\frac{1}{2} F_{la';nm} + B_{pl;n} D_{pm}^{a'} + H_{l,nm}^{a'} \right) \lambda_{a'} \xi_n \xi_m \dot{\xi}_l, \tag{72}$$

$$(k) = -\omega_{a'} \omega_{b'} \left[\left(\frac{1}{4} F_{a'b';nm} + B_{lb';n} + \frac{1}{2} D_{ln}^{a'} \right) D_{lm}^{b'} + H_{b';nm}^{a'} \right] \lambda_{a'} \lambda_{b'} \xi_n \xi_m, \tag{73}$$

$$(l) = \omega_{a'}^2 H_{a';nm}^b i\bar{\eta}_{a'} \eta_b \xi_n \xi_m. \tag{74}$$

There is also a coupling term between λ and the collective operators I (cf. Eq.(36)). When integrating over the fields q, λ and the ghosts, the I's behave like (non-commuting) sources for λ. The corresponding vertex is shown in Fig.2, and is equal to $-i\lambda_{a'} I_{a'} \omega_{a'} / \sqrt{\Im_{a'}}$.

A. Two-loop Corrections

The two loops correction to the partition function (excluding the terms depending on $I_{a'}$ which we evaluate below) is given by the diagrams of Fig.3. The values of the diagrams are,

$$(a) = \frac{1}{8} \left[\left(\omega_{\bar{m}}^2 B_{\bar{m}\bar{m};\bar{n}} - A_{\bar{m}\bar{m}\bar{n}} \right) G_{\omega_{\bar{m}}}(0) + \omega_{\bar{n}}^2 D_{\bar{n}a'}^{a'} G_{\omega_{a'}}(0) \right]^2 g(\omega_{\bar{n}}) \tag{75}$$

$$(b) = \frac{\beta}{2} \omega_{a'}^2 \left(B_{a'm;n} + D_{mn}^{a'} \right)^2 G_{\omega_{a'}}(0) G_{\omega_n}(0)$$

$$- \frac{\beta}{2} \omega_m^2 B_{mp;n}^2 G_{\omega_m}(0) G_{\omega_n}(0)$$

$$- \frac{1}{2} \frac{\omega_{c'}^2}{\Im_{c'}} C_{c'\bar{b}}^{a'} C_{c'a'}^{\bar{b}} g(\omega_{c'}, \dot{\omega}_{a'})$$

$$+ \frac{1}{4} \omega_{a'}^2 \omega_{b'}^2 \left[\left(B_{a'b';n} + D_{b'n}^{a'} + D_{a'n}^{b'} \right)^2 - 2 D_{b'n}^{a'} D_{a'n}^{b'} \right] g(\omega_{a'}, \omega_{b'}, \omega_n)$$

$$- \frac{1}{2} \omega_{a'}^2 \omega_m^2 \left(B_{a'm;n} + D_{mn}^{a'} \right)^2 g(\omega_{a'}, \omega_m, \omega_n)$$

$$+ \frac{1}{4} \left(\omega_m^2 \omega_p^2 B_{mp;n}^2 + \frac{1}{3} A_{nmp}^2 \right) g(\omega_n, \omega_m, \omega_p)$$

$$- \frac{1}{2} \omega_{a'}^2 \left(B_{a'm;n} + D_{mn}^{a'} \right) \left(B_{a'n;m} + D_{nm}^{a'} \right) g(\omega_{a'}, \dot{\omega}_m, \dot{\omega}_n)$$

$$+ \frac{1}{2} \left(\omega_n^2 B_{mn;p} B_{pn;m} + B_{mp;n} A_{mpn} \right) g(\omega_n, \dot{\omega}_m, \dot{\omega}_p) \tag{76}$$

$$(c) = \frac{\beta}{4} \omega_{\bar{n}}^2 F_{\bar{n}\bar{n};ll} G_{\omega_{\bar{n}}}(0) G_{\omega_l}(0)$$

$$- \beta \omega_{a'}^2 \left(B_{la';n} + \frac{1}{2} D_{ln}^{a'} \right) D_{ln}^{a'} G_{\omega_{a'}}(0) G_{\omega_n}(0)$$

$$- \frac{\beta}{8} E_{nnll} G_{\omega_n}(0) G_{\omega_l}(0) \tag{77}$$

where the functions $g(\omega_{\bar{n}})$, $g(\omega_{\bar{n}}, \omega_{\bar{l}}, \omega_{\bar{m}})$, $g(\omega_{\bar{l}}, \dot{\omega}_{\bar{n}}, \dot{\omega}_{\bar{m}})$ and $g(\omega_n, \dot{\omega}_m)$ are defined in the Appendix. The contributions proportional to $\delta(0)$ coming from the exponentiation of $\prod_\tau \sqrt{g}$ (cf. Eq.(29)) and from the propagator $\langle \langle \hat{T} \dot{\xi}_n \dot{\xi}_n \rangle \rangle$ cancel exactly (as expected) and are not listed.

Summing up all diagrams and using the relations given in the Appendix to simplify the result we obtain:

$$Z^{(2)} = \frac{1}{4} \left(\omega_{\bar{m}}^2 \omega_{\bar{l}}^2 B_{\bar{m}\bar{l};\bar{n}}^2 + \frac{1}{3} A_{\bar{n}\bar{m}\bar{l}}^2 \right) g(\omega_{\bar{n}}, \omega_{\bar{l}}, \omega_{\bar{m}})$$

$$+ \frac{1}{2} \left(B_{\bar{m}\bar{n};\bar{l}} A_{\bar{m}\bar{n}\bar{l}} + \omega_{\bar{l}} B_{\bar{m}\bar{l};\bar{n}} B_{\bar{n}\bar{p};\bar{m}} \right) g(\omega_{\bar{l}}, \dot{\omega}_{\bar{n}}, \dot{\omega}_{\bar{m}})$$

$$+\frac{1}{8}\left[\left(\omega_{\bar{m}}^2 B_{\bar{m}\bar{m};\bar{n}} - A_{\bar{m}\bar{m}\bar{n}}\right)G_{\omega_{\bar{m}}}(0)\right]^2 g(\omega_{\bar{n}})$$

$$+\left(-\frac{1}{8}E_{\bar{n}\bar{n}\bar{m}\bar{m}} + \frac{1}{4}\omega_{\bar{n}}^2 F_{\bar{n}\bar{n};\bar{m}\bar{m}} - \frac{1}{2}\omega_{\bar{n}}^2 B_{\bar{n}l;\bar{m}}^2\right)G_{\omega_{\bar{n}}}(0)G_{\omega_{\bar{m}}}(0)$$

$$-\frac{\beta}{8}\left(2B_{a'\bar{m}\bar{n}} + D_{\bar{m}\bar{n}}^{a'}\right)\left[D_{\bar{n}\bar{m}}^{a'} + 4D_{\bar{m}\bar{n}}^{a'}\omega_{\bar{m}}^2\beta G_{\omega_{\bar{n}}}(0)G_{\omega_{\bar{m}}}(0)\right]$$

$$+\frac{\beta}{8}D_{a'c'}^{b'}D_{b'c'}^{a'} + \frac{\beta}{8}D_{b'\bar{m}}^{a'}D_{a'\bar{m}}^{b'} - \frac{\beta}{8}B_{a'b';\bar{m}}D_{\bar{m}b'}^{a'} - \frac{\beta}{8\Im_{c'}}C_{c'\bar{b}}^{a'}C_{c'a'}^{\bar{b}}, \tag{78}$$

where we see that all spurious frequencies have disappeared, as it should. It can be checked that in simple cases the known results are reobtained, as for example the $\lambda\phi^4$ kink in $1+1$ dimensions [22].

B. Collective Hamiltonian

The corrections involving the collective momenta I_a can be evaluated in a similar way as in the previous section. However, as is done in perturbation theory to a degenerate level the result is best expressed as an effective Hamiltonian acting over collective variables. To tree level the diagrams involved are the ones of Fig.4, which are of $\mathcal{O}(\Im^{-1})$

$$H_{\text{coll}} = \frac{1}{2\Im_{a'}}I_{a'}^2 + \mathcal{O}(\Im^{-2}). \tag{79}$$

To one-loop the diagrams are the ones of Fig.5. They give

$$(\text{a}) = \frac{\{I_{a'}, I_{b'}\}}{\sqrt{\Im_{a'}\Im_{b'}}}\left\{\frac{1}{4}\left(B_{a'm;n} + D_{mn}^{a'}\right)\left(B_{b'm;n} + D_{mn}^{b'}\right)\times\right.$$

$$\times\left[-\beta G_{\omega_{\bar{n}}}(0) + \omega_m^2 g(\omega_n, \omega_m)\right]$$

$$+\frac{1}{4}\left(B_{a'm;n} + D_{mn}^{a'}\right)\left(B_{b'n;m} + D_{nm}^{b'}\right)g(\dot{\omega}_n, \dot{\omega}_m)$$

$$\left.-\frac{1}{4}\left(B_{a'c';n} + D_{c'n}^{a'} + D_{a'n}^{c'}\right)\left(B_{b'c';n} + D_{c'n}^{b'} + D_{b'n}^{c'}\right)\omega_{c'}^2 g(\omega_n, \omega_{c'})\right\}$$

$$-\frac{\beta I_{a'}I_{b'}}{\Im_{a'}\Im_{b'}}C_{b'\bar{c}}^{d'}C_{a'd'}^{\bar{c}}G_{\omega_{d'}}(0), \tag{80}$$

$$(\text{b}) = \frac{\{I_{a'}, I_{b'}\}}{8\sqrt{\Im_{a'}\Im_{b'}}}\left(B_{a'b';\bar{n}} + 2D_{b'\bar{n}}^{a'}\right)\times$$

$$\times\left[\left(\omega_{\bar{m}}^2 B_{\bar{m}\bar{m};\bar{n}} - A_{\bar{m}\bar{m}\bar{n}}\right)G_{\omega_{\bar{m}}}(0) + \omega_{\bar{n}}^2 D_{\bar{n}c'}^{c'}G_{\omega_{c'}}(0)\right]g(\omega_{\bar{n}}^2), \tag{81}$$

$$(\text{c}) = \frac{\beta\{I_{a'}, I_{b'}\}}{\sqrt{\Im_{a'}\Im_{b'}}}G_{\omega_n}(0)\left[\frac{1}{8}F_{a'b';nn} + \frac{1}{2}B_{la';n}D_{ln}^{b'} + \frac{1}{4}D_{ln}^{a'}D_{ln}^{b'} + \frac{1}{4}H_{b';nn}^{a'}\right]. \tag{82}$$

There is a further contribution from the vertex $-i\lambda_{a'}I_{a'}\omega_{a'}/\sqrt{\Im_{a'}}$ (Fig.2) at fourth order. Such contribution correspond to disconnected diagrams and should vanish but for the fact that the generators $I_{a'}$ do not conmute. Using perturbation theory to a degenerate level we obtain

$$
-\frac{I_{a'}I_{a'}I_{b'}I_{b'}}{\Im_{a'}\Im_{b'}}\beta\frac{1}{2}G_{\omega_{a'}}(0) + \frac{I_{a'}I_{b'}I_{a'}I_{b'}}{\Im_{a'}\Im_{b'}}\frac{1}{8}g(\dot{\omega}_{a'},\dot{\omega}_{b'}) - \frac{I_{a'}I_{b'}I_{b'}I_{a'}}{\Im_{a'}\Im_{b'}}\left(\frac{1}{8}g(\dot{\omega}_{a'},\dot{\omega}_{b'}) - \frac{1}{2}\beta G_{\omega_a}(0)\right),
$$

(83)

which clearly vanishes if the I's commute. As was discussed in Sec.V, the $I_{\bar{a}}$'s can be identified with intrinsic operators, so that the contributions proportional to them should be included in the intrinsic correction to $\mathcal{O}(\Im^{-2})$ (three-loop diagrams). Taking this into account, the contribution of (83) to the collective hamiltonian reads

$$
\frac{I_{a'}I_{b'}}{\Im_{a'}\Im_{b'}}\frac{1}{2}\beta G_{\omega_{c'}}(0)C_{c'b'}{}^{\bar{d}}C_{c'\bar{d}}{}^{a'}.
$$

(84)

The one-loop correction to the effective collective Hamiltonian (79) can be obtained as $-1/\beta$ times the sum of all diagrams plus (84). Using the identities of the Appendix it reads:

$$
\begin{aligned}
H_{\text{coll}}^{1-\text{loop}} = &-\frac{\{I_{a'},I_{b'}\}}{\sqrt{\Im_{a'}\Im_{b'}}}G_{\omega_{\bar{n}}}(0)\left[\frac{1}{8}F_{a'b',\bar{n}\bar{n}} + \frac{1}{4}H_{b';\bar{n}\bar{n}}^{a'}\right.\\
&+\frac{1}{8\omega_{\bar{n}}^2}\left(B_{a'b';\bar{n}} + 2D_{b'\bar{n}}^{a'}\right)\left(\omega_{\bar{m}}^2 B_{\bar{m}\bar{m};\bar{n}} - A_{\bar{m}\bar{m}\bar{n}}\right)\\
&-\frac{1}{4}B_{a'm;\bar{n}}B_{b'm;\bar{n}} - \frac{1}{4}D_{a'\bar{n}}^{c'}D_{b'\bar{n}}^{c'}\\
&\left.-\frac{1}{2}\left(B_{a'c';\bar{n}}D_{b'\bar{n}}^{c'} + D_{c'\bar{n}}^{a'}D_{b'\bar{n}}^{c'}\right)\right]\\
&-\frac{\{I_{a'},I_{b'}\}}{\beta\sqrt{\Im_{a'}\Im_{b'}}}\left[\frac{1}{4}\omega_{\bar{m}}^2\left(B_{a'\bar{m};\bar{n}} + D_{\bar{m}\bar{n}}^{a'}\right)\left(B_{b'\bar{m};\bar{n}} + D_{\bar{m}\bar{n}}^{b'}\right)g(\omega_{\bar{n}},\omega_{\bar{m}})\right.\\
&\left.+\frac{1}{4}\left(B_{a'\bar{m};\bar{n}} + D_{\bar{m}\bar{n}}^{a'}\right)\left(B_{b'\bar{n};\bar{m}} + D_{\bar{n}\bar{m}}^{b'}\right)g(\dot{\omega}_{\bar{n}},\dot{\omega}_{\bar{m}})\right].
\end{aligned}
$$

(85)

Again it is seen to be independent of the arbitrary parameters $\omega_{a'}$, which gives a useful test of the calculation. It has a dependence in β because it comprises the collective Hamiltonian asociated with all vibrational states. If, for example, we need the collective energies associated with the ground state, we just extract the $\beta \to \infty$ limit. If we need that of the $\omega_{\bar{n}}$ level, we look for the $e^{-\beta\omega_{\bar{n}}}$ dependent term.

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The above result gives the correction to the inertia parameters $\Im_{a'}$. To obtain the correction to $\mathcal{O}(\Im^{-2})$ to the energy levels the three-loop intrinsic diagrams should be computed.

VII. CONCLUSIONS

In this paper we have applied a Lagrangian (antifield) BRST formalism to the quantization of collective coordinates of a model describing the motion of a particle on a Riemannian manifold under a scalar potential.

To perform a semiclassical expansion for such a theory the minima of the potential must be found, and the potential must be expanded around them. This yields a set of harmonic oscillators plus higher order terms which are treated in perturbation theory. As is well known, this scheme fails if the potential has a manifold of minima. In such case there are directions with zero restoring force which spoils the perturbation theory with infrared divergences. The classical solution breaks a symmetry which is quantically unbroken.

In order to restore the symmetry and to cure the infrared divergences we introduced explicitly collective coordinates, as time dependent parameters of the symmetry transformations of the action broken by the classical solution. The group of symmetries of the action was considered to be in general non-abelian.

The resulting gauge theory was quantized using the antifield formalism. A gauge-fixed BRST invariant action was obtained. It was also shown that the same result can be found by a hamiltonian BRST procedure.

We took advantage of finite temperature techniques to calculate the partition function. It was shown that a suitable choice of the gauge-fixing functions decouples the fields at quadratic level and provides non-zero spurious frequencies to the zero modes.

We calculated the intrinsic partition function at two-loops, and the collective hamiltonian at one-loop. It was shown that these results are independent of the gauge-fixing parameters, as expected.

The model considered in this paper encompasses, in the case of an infinite number of

coordinates, soliton models, as for example the Skyrme model. However, the application of our results to such a system would involve the numerical task of determining the normal modes of the quadratic action, as well as the solution of ultraviolet problems.

ACKNOWLEDGMENTS

APPENDIX:

When expanding the action in terms of powers of the normal modes it is useful to introduce a number of coefficients whose definitions we collect below:

$$A_{nml} = \bar{V}_{stu}\psi_n^s\psi_m^t\psi_l^u, \tag{A1}$$

$$B_{nm;l} = \bar{g}_{st,u}\psi_n^s\psi_m^t\psi_l^u, \tag{A2}$$

$$D_{nm}^{a'} = \Im_{a'}^{-\frac{1}{2}}\bar{g}_{st}\partial_{a'v}R^s(\bar{q},0)\psi_n^t\psi_m^v, \tag{A3}$$

$$D_{nm}^{\bar{a}} = \bar{g}_{st}\partial_{\bar{a}v}R^s\psi_n^t\psi_m^v, \tag{A4}$$

$$E_{nmlk} = \bar{V}_{stuv}\psi_n^s\psi_m^t\psi_l^u\psi_k^v, \tag{A5}$$

$$F_{nm;lk} = \bar{g}_{st,uv}\psi_n^s\psi_m^t\psi_l^u\psi_k^v, \tag{A6}$$

$$H_{m;nl}^{a'} = \Im_{a'}^{-\frac{1}{2}}g_{st}\partial_{auv}R^s(\bar{q},0)\psi_m^t\psi_n^u\psi_l^v. \tag{A7}$$

These coefficients are related to each other by the fact that the action (21) is gauge invariant. To find all relations which are useful to simplify the calculations we differentiate both sides of Eqs.(3-4) with respect to α_a and evaluate them at $\alpha_a = 0$:

$$\partial_s V[R(q,0)]\partial_a R^s = 0, \tag{A8}$$

$$\partial_a\partial_s R^u(q,0)g_{ut} + \partial_a\partial_t R^v(q,0)g_{sv} + \partial_a R^w(q,0)g_{st,w} = 0, \tag{A9}$$

where we have used that $\partial_s R^t(q,0) = \delta_s^t$.

Expanding these equations around the classical minimum the desired relations are obtained:

$$E_{mnla'} = -A_{kmn}D_{kl}^{a'} - A_{kml}D_{kn}^{a'} - A_{knl}D_{km}^{a'}$$
$$-\tilde{\omega}_m^2 H_{m;nl}^{a'} - \tilde{\omega}_n^2 H_{n;ml}^{a'} - \tilde{\omega}_l^2 H_{l;mn}^{a'}, \tag{A10}$$

$$A_{a'nm} = -\tilde{\omega}_n^2 D_{nm}^{a'} - \tilde{\omega}_m^2 D_{mn}^{a'}, \tag{A11}$$

$$B_{nm;a'} = -D_{mn}^{a'} - D_{nm}^{a'}, \tag{A12}$$

$$F_{nl;ma'} = -B_{kl;m}D_{kn}^{a'} - B_{nk;m}D_{kl}^{a'} - B_{nl;k}D_{km}^{a'}$$
$$-H_{l;nm}^{a'} - H_{n;ml}^{a'}, \tag{A13}$$

$$D_{\bar{n}b'}^{\bar{a}} = D_{b'\bar{n}}^{\bar{a}} = 0 \tag{A14}$$

$$D_{c'b'}^{\bar{a}} = -\sqrt{\frac{\Im_{c'}}{\Im_{b'}}}C_{\bar{a}b'}{}^{c'}, \tag{A15}$$

$$C_{\bar{a}c'}{}^{b'} = -\frac{\Im_{c'}}{\Im_{b'}}C_{\bar{a}b'}{}^{c'}, \tag{A16}$$

where, for brevity, we define $\tilde{\omega}_{\bar{n}} = \omega_{\bar{n}}$, $\tilde{\omega}_{a'} = 0$. Further relations can be obtained using the commutation relations (6):

$$D_{nb'}^{a'} - D_{na'}^{b'} = 0 \tag{A17}$$

$$D_{mn}^{a'}D_{nl}^{b'} - D_{mn}^{b'}D_{nl}^{a'} = \sqrt{\frac{\Im_{c'}}{\Im_{a'}\Im_{b'}}}C_{b'a'}{}^{c'}D_{ml}^{c'}$$
$$+\sqrt{\frac{1}{\Im_{a'}\Im_{b'}}}C_{b'a'}{}^{\bar{c}}D_{ml}^{\bar{c}} + H_{m;la'}^{b'} - H_{m;lb'}^{a'} \tag{A18}$$

It is also useful to define some functions obtained integrating the thermal propagator:

$$g(\omega_n) = \int_0^\beta d\tau G_{\omega_n}(\tau) = \frac{\beta}{\omega_n^2} \tag{A19}$$

$$g(\omega_n, \omega_m, \omega_l) = \int_0^\beta d\tau G_{\omega_n}(\tau)G_{\omega_m}(\tau)G_{\omega_l}(\tau)$$
$$= \beta\left[\omega_n\left(\omega_n^2 - \omega_m^2 - \omega_l^2\right)\coth\left(\frac{\beta\omega_m}{2}\right)\coth\left(\frac{\beta\omega_l}{2}\right)\right.$$
$$+\omega_m\left(\omega_m^2 - \omega_n^2 - \omega_l^2\right)\coth\left(\frac{\beta\omega_n}{2}\right)\coth\left(\frac{\beta\omega_l}{2}\right)$$

$$+\omega_l \left(\omega_l^2 - \omega_m^2 - \omega_n^2\right) \coth\left(\frac{\beta\omega_m}{2}\right) \coth\left(\frac{\beta\omega_n}{2}\right) + 2\omega_n\omega_m\omega_l \Bigg] \Big/$$

$$\left[4\omega_n\omega_m\omega_l \left(\omega_n + \omega_m + \omega_l\right)\left(\omega_n + \omega_m - \omega_l\right)\left(\omega_n - \omega_m + \omega_l\right)\right.$$

$$\left.\left(\omega_n - \omega_m - \omega_l\right)\right] \tag{A20}$$

$$g(\omega_n, \dot\omega_m, \dot\omega_l) = \int_0^\beta d\tau\, G_{\omega_n}(\tau)\partial_\tau G_{\omega_m}(\tau)\partial_\tau G_{\omega_l}(\tau)$$

$$= \beta \Bigg[\omega_n\left(\omega_n^2 - \omega_m^2 - \omega_l^2\right) + \omega_m\left(\omega_m^2 - \omega_n^2 + \omega_l^2\right) \coth\left(\frac{\beta\omega_m}{2}\right) \coth\left(\frac{\beta\omega_n}{2}\right)$$

$$+\omega_l\left(\omega_l^2 - \omega_n^2 + \omega_m^2\right) \coth\left(\frac{\beta\omega_l}{2}\right) \coth\left(\frac{\beta\omega_n}{2}\right) +$$

$$+2\omega_n\omega_m\omega_l \coth\left(\frac{\beta\omega_m}{2}\right) \coth\left(\frac{\beta\omega_l}{2}\right)\Bigg] \Big/$$

$$4\omega_n\left(\omega_n + \omega_m + \omega_l\right)\left(\omega_n + \omega_m - \omega_l\right)\left(\omega_n - \omega_m + \omega_l\right)\left(\omega_n - \omega_m - \omega_l\right) \tag{A21}$$

$$g(\omega_n, \omega_m) = \int_0^\beta d\tau\, G_{\omega_n}(\tau)G_{\omega_m}(\tau)$$

$$= \frac{\beta}{2\omega_n\omega_m\left(\omega_n^2 - \omega_m^2\right)} \left[\omega_n \coth\left(\frac{\beta\omega_m}{2}\right) - \omega_m \coth\left(\frac{\beta\omega_n}{2}\right)\right] \tag{A22}$$

$$g(\dot\omega_n, \dot\omega_m) = \int_0^\beta d\tau\, \partial_\tau G_{\omega_n}(\tau)\partial_\tau G_{\omega_m}(\tau)$$

$$= \frac{\beta}{2\left(\omega_n^2 - \omega_m^2\right)} \left[\omega_n \coth\left(\frac{\beta\omega_n}{2}\right) - \omega_m \coth\left(\frac{\beta\omega_m}{2}\right)\right] \tag{A23}$$

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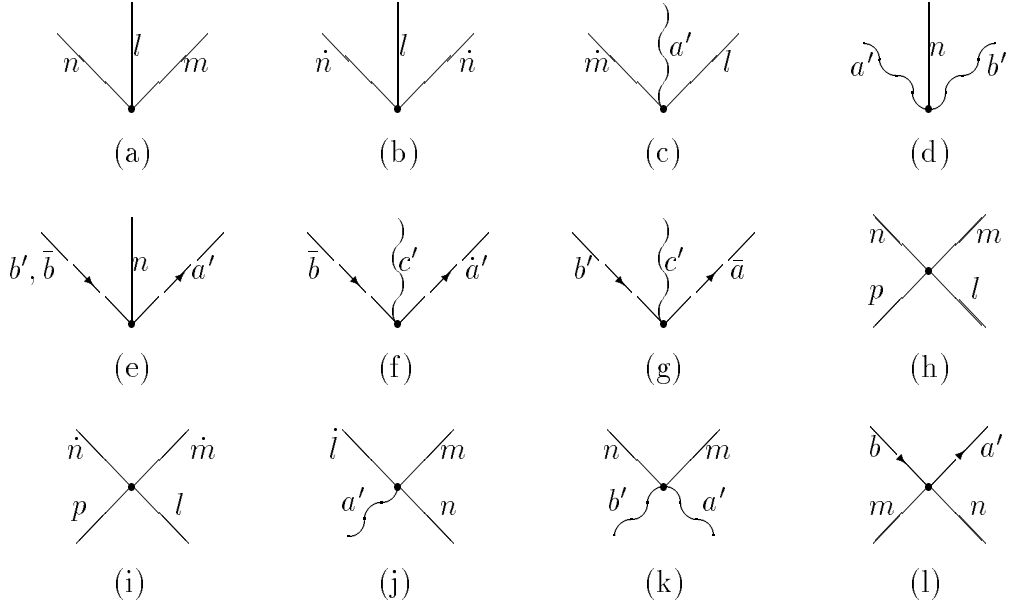[29] I.Gerstein, R.Jackiw, B.Lee, S.Weinberg, Phys.Rev.D **3**, 2486 (1971).

FIGURES

FIG. 1. Third and fourth order vertices from $L_{\text{BRST}}^{(3)}$ and $L_{\text{BRST}}^{(4)}$.
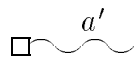
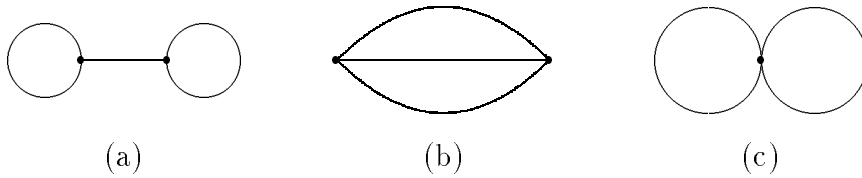FIG. 2. Collective-intrinsic coupling vertex.

FIG. 3. Two-loops diagrams involved in the correction to the partition function.

FIG. 4. Collective energy to lowest order.

FIG. 5. First correction to the collective energy.

FIG. 1. Third and fourth order vertices from $L_{\mathrm{BRST}}^{(3)}$ and $L_{\mathrm{BRST}}^{(4)}$.

FIG. 2. Collective-intrinsic coupling vertex.

(a)   (b)   (c)

FIG. 3. Two-loops diagrams involved in the correction to the partition function.
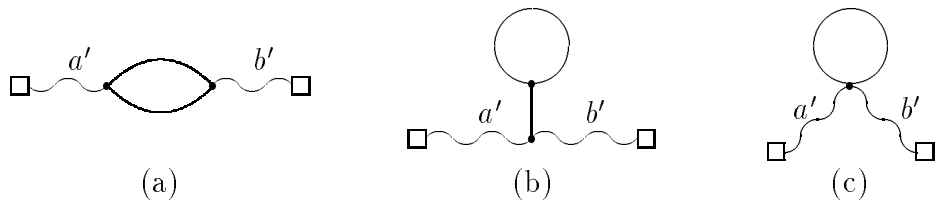
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